Application of nonlinear stability analysis in railway vehicle industry

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Abstract This paper deals with the use of nonlinear calculations and bifurcation analysis when investigating running stability during vehicle design and development in the rolling stock industry. Typical methods used for stability analysis in industrial applications are introduced, computation of bifurcation diagram presented and the influence of nonlinearities of the vehicle/track system on the type of Hopf bifurcation investigated. The relationship between the bifurcation diagram and the assessment of safety risk and the dynamic behaviour is discussed.

1. Introduction

A self-excited, sustained oscillation of wheelsets with conventional solid axles is a classic problem of railway vehicle dynamics. It is called hunting or instability by railway engineers. The frequency of such waving motion of wheelsets and bogies is related to the wheel/rail contact geometry. Equivalent conicity is applied as a simplified parameter in order to describe the wheel/rail contact geometry in railway practice. The equivalent conicity can vary to a large degree and therefore plays a significant role in the stability assessment of railway vehicles.

If the wheel/rail contact conditions lead to a bogie motion with a low frequency, approaching the vehicle carbody natural frequency, the possibility of considerable interaction may arise, leading to a limit cycle oscillation during which the amplitude of the car body is large relative to that of the wheelsets. In this case we refer to carbody instability (primary instability) or carbody hunting. If only the wheelsets and bogies or running gears are involved in the limit cycle oscillation, we refer to bogie instability (secondary instability) or bogie hunting. In modern vehicles carbody instability leads to a deterioration of lateral running behaviour, as well as ride comfort degradation without exceeding the safety criteria. A wheel/rail contact geometry characterized by high conicity typically limits the maximum permissible speed with respect to bogie hunting, i.e. running safety.

The necessity of stability investigations was only slowly recognized during the mid-twentieth century. A theoretical comprehension of railway vehicle stability

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came into being as a result of studies on linearised models; see e. g. [1] for details. At a later date, nonlinearities of the wheel/rail combination were also taken into consideration, see [2] and [3] for further references.

The publications dealing with nonlinear stability assessment of railway vehicles often apply simplified models, conical or theoretical wheel profiles and theoretical rail profiles. No systematic study about the influence of nonlinearities on the stability and bifurcation behaviour of large vehicle models has been published yet. Considering complex systems of the vehicle/track and a large variation of wheel/rail contact geometries and friction conditions in railway service, the question appears how far are the conclusions from the published investigations valid for the industrial applications?

This article deals with use of nonlinear calculations and bifurcation analysis when investigating running stability during vehicle design and development in the rolling stock industry. It is organised as follows. Methods typically used for stability analysis in industrial application are introduced in Chapter 2. In Chapter 3, the bifurcation analysis is presented and the impact of the nonlinearities of the vehicle/track system on the bifurcation diagram explained. Chapter 4 discusses the relationship between the bifurcation diagram and the assessment of safety risk and the vehicle's dynamic behaviour.

2. Assessment of the running stability in railway industry

Stability analysis constitutes the most diversified part of vehicle dynamics due to the various possible methods, the wide range of input conditions and different assessment criteria. In spite of the vehicle/track system being always nonlinear, both nonlinear as well as linear calculations are applied for the stability assessment. In the linearized stability assessment, the contact of the wheelset and track is linearized differently to the other coupling elements. The quasi-linearization of wheel/rail contact, in which linearized wheel/rail parameters are computed for the specified wheelset lateral movement amplitude, is the standard method implemented in simulation tools used in railway vehicle engineering. Comparison of linearized and nonlinear stability assessment has been presented by the author in [4].

The nonlinear methods of stability assessment using computer simulations have been compared and discussed by the author in [3]. These can be classified depending on the track alignment used of:

- ideal track (no irregularity)
- real track with track irregularity (measured irregularities)
- combination of track disturbance followed by a section of ideal track, whereby the track disturbance can be represented by
  - single lateral disturbance
  - track section with irregularity.

Another classification can be introduced in relation to the assessment criteria of:

- decay of oscillations
- limit values specified for testing for the acceptance of running characteristics of railway vehicles in EN 14363 [5].

A lateral displacement of wheelsets is usually used to prove the decay of self-excited oscillations of a railway vehicle. Displacements of other bodies (bogie frame, carbody) can gain additional information to distinguish between the hunting of bogie or carbody.

The criteria used during the testing of vehicles for the acceptance of running characteristics are:

- forces between wheelset and track (sliding rms-value of sum of guiding forces) as specified for normal measuring method according to EN 14363 [5]
- lateral acceleration on the bogie frame (sliding rms-value) as specified for simplified measuring method according to EN 14363 [5].

There are pros and cons for all methods mentioned. The three most used methods are illustrated by examples of safety assessment, considering wheel/rail friction coefficient of 0.4 and a high equivalent conicity of 0.6 for the wheelset amplitude of 3 mm.

Method 1: Figure 1 shows the wheelsets lateral displacement as a result of simulation on ideal track, starting from a limit cycle at high speed and reducing the speed slowly. The speed at which the oscillation disappears is then the nonlinear critical speed [6].

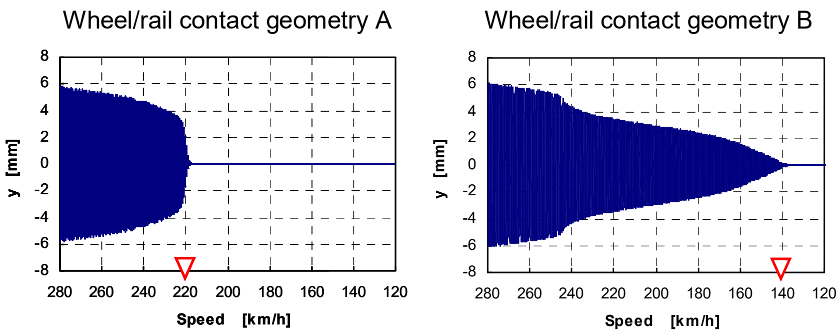

Fig. 1. Simulations of run on ideal track with speed decreasing by 4 km/h per second.

Method 2: Figure 2 shows simulations of the reaction to a single lateral disturbance with amplitude of 8 mm and a span of 10 m, followed by an ideal track, for the variation of vehicle speed.

Method 3: Figure 3 presents simulations of a run on track with measured irregularities and analyses of the criteria according to EN 14363 for wheel/rail contact geometry A.

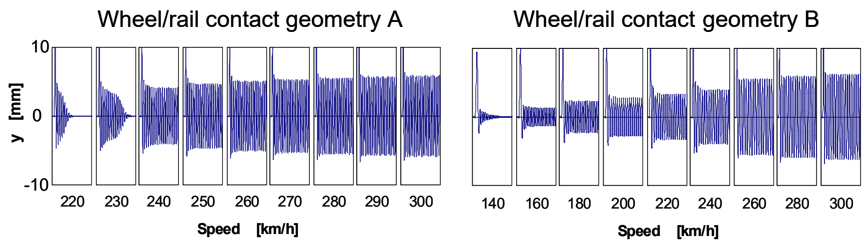

Fig. 2. Simulations of wheelset reaction on a single lateral excitation on ideal track.

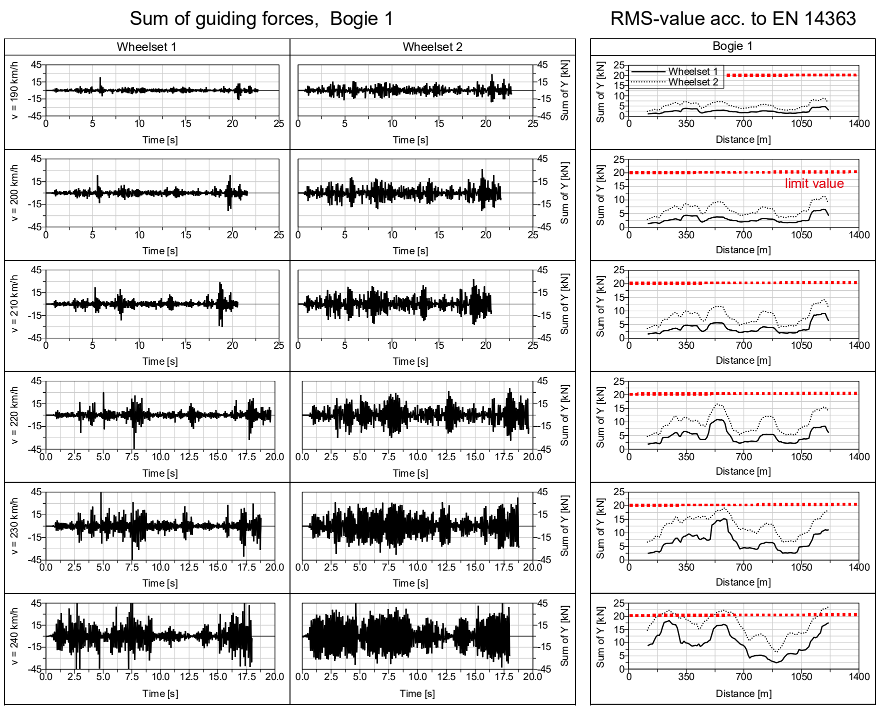

Fig. 3. Stability assessment based on simulations of vehicle acceptance tests.

While the first method allows an unambiguous assessment of critical speed, it is rather rarely used as it requires a long simulation time. The second method is often used because of simple handling and short simulation times. Likewise, the third method is often applied because of the easy possibility of comparison with vehicle test results.

The examples in Figure 1 demonstrate different critical speeds and different behaviour for the same vehicle with the same equivalent conicity for the wheelset amplitude of 3 mm as specified for vehicle acceptance tests [5]. There is abrupt wheelset stabilization in the first example, whereas in the second example the amplitude of the limit cycle slowly reduces with decreasing speed.

Differing behaviour can be observed also in the examples in Figure 2. The differences result from the nonlinearities of the investigated system. A prominent feature of nonlinear dynamical systems is the possible dependence of their long-time behaviour on the initial conditions, leading to the existence of multiple solutions.

The methods discussed so far can however only identify one solution. Furthermore, differing procedures and criteria for the stability assessment in railway applications can lead to different conclusions, because a limit cycle oscillation with a rather small amplitude will not necessarily lead to exceedance of the stability limit during vehicle testing. This can be seen in Figure 4 on the analysis of the vehicle behaviour on an ideal smooth track behind a single disturbance. An assessment of nonlinear dynamical systems with respect to the influence of one or more system parameters on existence of multiple solutions can be carried out by bifurcation analysis, which will be discussed in the next chapter.

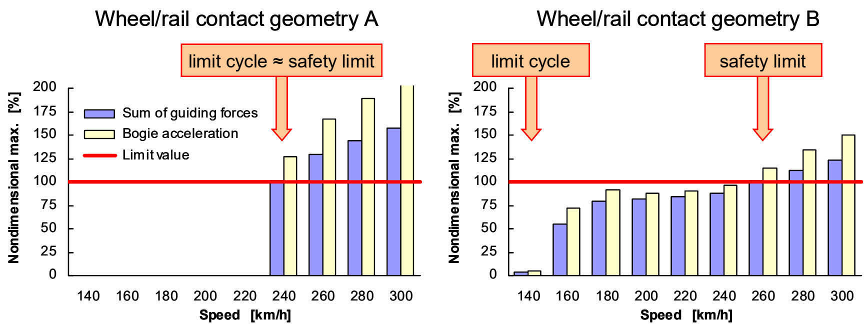

Fig. 4. Comparison of stability assessment based on the occurrence of a limit cycle and the assessment according to the safety limits specified for measurements in EN 14363.

3. Bifurcation analysis of the system vehicle/track

The usual way to present the bifurcation phenomenon is a bifurcation diagram [2]. When analysing the stability of railway vehicles, the bifurcation diagram displays the amplitude of the limit cycle (typically lateral wheelset displacement) as a function of speed. For some systems, the bifurcation diagram can be very complex including quasi-periodic or chaotic motion. Considering the main shape of the diagram, we can distinguish between the subcritical and supercritical Hopf bifurcation; see Figure 5 [2], [3]. In case of subcritical bifurcation there is a speed range at which the solution can “jump” between a stable damped movement and a limit cycle depending on the excitation amplitude.

The calculation of the bifurcation diagram can proceed by a path following method (continuation) or by a set of numerical simulations. A software tool PATH for the continuation-based bifurcation analysis has been developed at the Technical University of Denmark [7]. It uses a mixture of integration in time and Newton iteration to find the periodic solutions. The code starts with the trivial solution that is known to be asymptotically stable at sufficiently low speed. The speed is then increased in small steps and the solution is followed for each value of the speed. When a bifurcation point is reached, the path to be followed is chosen in the phase-parameter space.

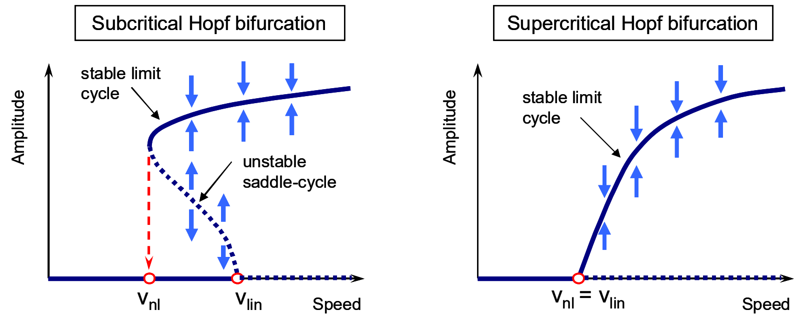

Fig. 5. Bifurcation diagram with subcritical and supercritical Hopf bifurcation; rows show the feedback of the nonlinear system.

The integration of the software tool PATH with commercial MBS-software SIMPACK has been developed and described by Schupp [8]. However, this tool is not commercially available as a part of SIMPACK software package yet. Furthermore, the method is not straight forward. As stated in [8], external time integration is required to generate an initial estimation, because it is not possible to continue the unstable periodic solutions branching off from the Hopf bifurcation of the first branch.

Stichel [9] uses a rather straight forward method applying numerical simulation. A run over an initial lateral disturbance is simulated at a rather high speed. The simulation continues on undisturbed track until the oscillation of the vehicle has reached constant amplitude. The vehicle speed is reduced and a new simulation started with initial values from the previous simulation. This is repeated until the oscillating solution disappears.

A set of numerical simulations is also applied in the investigations presented in this article. A run over a single lateral disturbance with a span of 10 m is simulated and the amplitude of the limit cycle after a few seconds, behind the transition process, taken in the bifurcation diagram. This is repeated for a set of speeds including those leading to limit cycle oscillations. As first, a large disturbance with an amplitude of 8 mm was used to identify nonlinear critical speed. Then, a set of simulations with speed variation is repeated applying a small disturbance with 0.5 mm amplitude. If the solution without oscillations appears for speeds higher than the nonlinear critical speed, a set of simulations with amplitude variation is started, to identify the amplitude for which the stable solution without oscillation changes to a limit cycle. This value is then taken as a point of the unstable branch for the considered speed in the bifurcation diagram.

To study the impact of the nonlinearities and non-smoothness of the system vehicle/track, we will distinguish between:

- nonlinearity and non-smoothness of wheel-rail contact
- nonlinearity and non-smoothness of vehicle model itself.

The effects of these two groups of nonlinearities on the shape of bifurcation diagram will be investigated in the following subchapters.

3.1 Wheel-rail contact nonlinearity

The influence of wheel/rail contact nonlinearity on railway vehicle behaviour at the stability limit can be seen in Figure 6. The simulations were carried out with a multi-body model of a four-car articulated vehicle in simulation tool SIMPACK. The model consists of 124 bodies and possesses 201 degrees of freedom (DOF). The diagrams display the results of the second wheelset of the leading bogie, where the limit cycle was first observed.

From the presented example as well as from the author's other studies, it can be observed, that the influence of the wheel/rail contact nonlinearity can be assessed with the help of the contact geometry functions used for the linearization of wheel/rail contact:

- Difference of rolling radii Δr of the left and right wheel in function of the wheelset lateral displacement
- Equivalent conicity λ in function of wheelset displacement amplitude.

Both examples in Figure 6 represent the same equivalent conicity for the amplitude of 3 mm. For the contact geometry A, there is a progressive increase of rolling radii difference in function of wheelset displacement and progressive equivalent conicity in function of wheelset amplitude. Abrupt limit cycle decay can be observed on the phase diagram of displacements of wheelset 1 and 2. There is a subcritical Hopf bifurcation in the bifurcation diagram of the wheelset 2. In contrast, for the contact geometry B there is strongly declining function of rolling radii difference and also strongly declining equivalent conicity function for amplitudes up to 4 mm (i.e. in the tread away from flange contact) due to large movement of the contact area for the wheelset displacement between 0 and 1 mm. A slow decrease of oscillations and a supercritical Hopf bifurcation can be observed for this wheel/rail contact geometry.

The different behaviour of railway vehicles on the contact geometry with the equivalent conicity function of "Type A" and "Type B" has been described for the first time in [10] and outlined more in detail in [3]. The nonlinearity of the contact geometry often determines the type of the Hopf bifurcation of railway vehicles. It contradicts the repeatedly presented statement that the bifurcation analysis of a railway vehicle always or mostly leads to the subcritical Hopf bifurcation, with the nonlinear critical speed lower than linear critical speed.

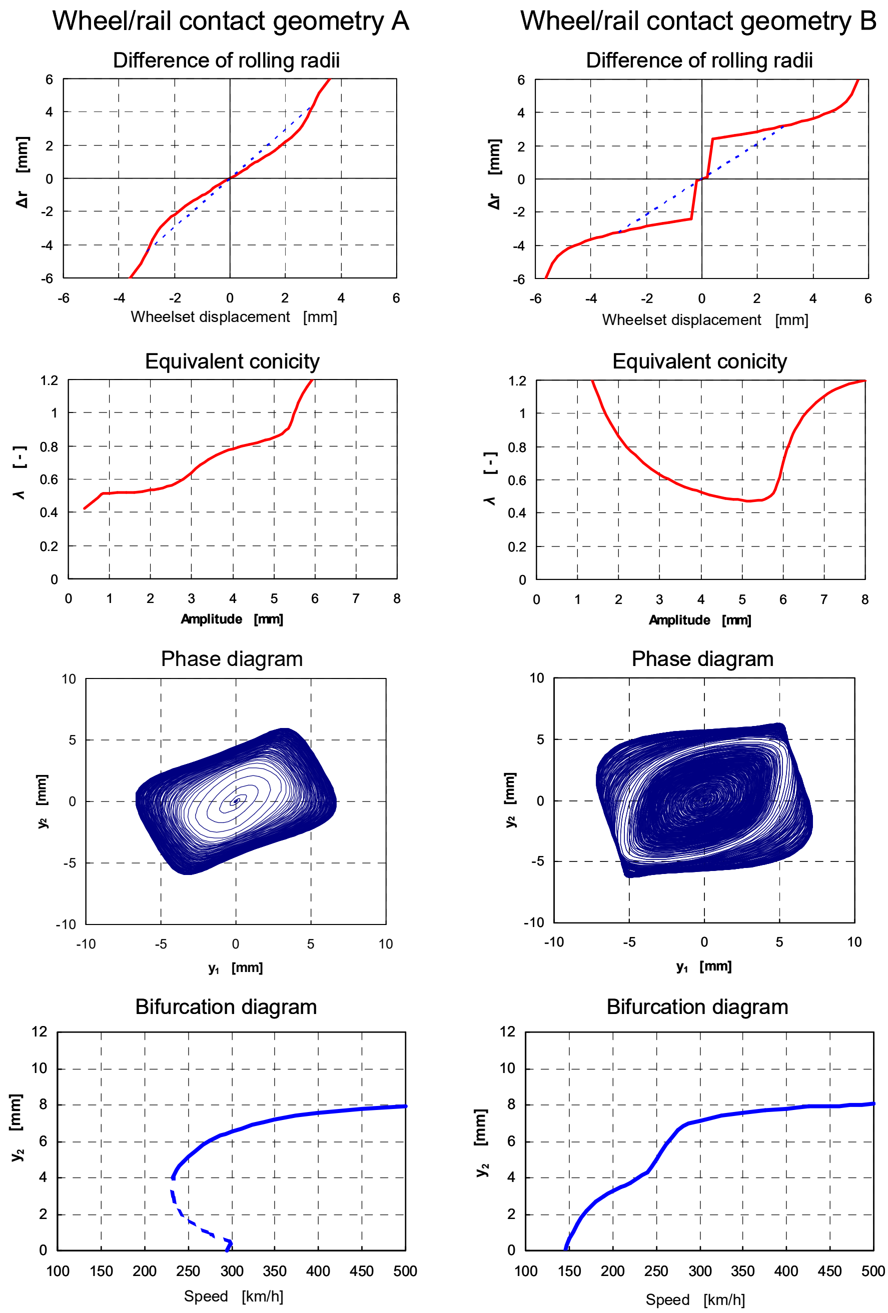

Fig. 6. Influence of wheel/rail contact nonlinearity on the behaviour of a railway vehicle at the stability limit on the example of two contact geometries with the same equivalent conicity for the wheelset amplitude of 3 mm.

In fact, the supercritical Hopf bifurcation can also occur with railway vehicles, and probably more frequently than supposed until now. This is because the contact geometry of “Type A” is related to a conical wheel profile which is the profile mostly used in theoretical studies. The change of the contact geometry due to wheel wear often leads to more conformal tread contact characterized by contact geometry of “Type B”, leading to the supercritical Hopf bifurcation with the nonlinear speed equal to the linear critical speed.

The nonlinear critical speed increases with reduction of wheel/rail friction coefficient, whereby the linear critical speed remains the same [3]. The subcritical Hopf bifurcation is less pronounced and can change to supercritical Hopf bifurcation for very low friction coefficients.

The described effects of wheel/rail contact conditions on bifurcation behaviour can be summarized as follows. The equivalent conicity value for the 3 mm amplitude characterizes the conicity level; a higher conicity leads to lower linear critical speed and vice versa. The slope of the equivalent conicity as a function of wheelset amplitude influences the type of Hopf bifurcation; a strongly decreasing conicity function for small amplitudes encourages supercritical Hopf bifurcation and vice versa. Similarly, the slope of creep force function influences linear critical speed, whereas the wheel/rail friction coefficient influences the form of Hopf bifurcation.

3.2 Nonlinearities of vehicle model

The nonlinearity of a vehicle model can supersede the effect of wheel/rail contact and change the type of Hopf bifurcation. Figure 7 shows as example the influence of nonlinear characteristic of yaw dampers on the bifurcation diagram of a double-decker coach with 39 bodies and 73 DOF, build in simulation tool SIMPACK. The nonlinearity of wheel/rail contact is still dominant, however, the implementation of yaw dampers with nonlinear characteristics leads to a change of the Hopf bifurcation for the wheel/rail contact geometry “Type A” in the left diagram. In contrast, on the right diagram (contact geometry “Type B”) there is always a supercritical Hopf bifurcation. Introducing the yaw dampers, oscillations with large amplitudes are suppressed into higher speeds, whereas small amplitudes below 2 mm remain present already for low speed.

Figure 8 shows as example the influence of nonlinear, non-smooth characteristic of yaw dampers on the bifurcation diagram of the same vehicle and the wheel/rail contact geometry A. The damping force of yaw dampers is nonlinear due to a strong slope reduction at the blow-off force. The yaw damper characteristic No. 3 in Figure 8 assumes a negligible force for a very small piston velocity, caused e.g. by a piston leakage. The variation of the blow-off force and the non-smooth characteristic of yaw damper, results to a change of Hopf bifurcation and to the variation of critical speed between 200 and 280 km/h.

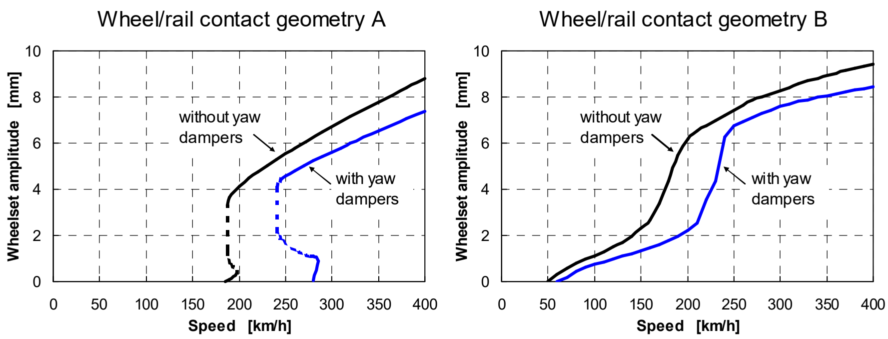

Fig. 7. Bifurcation diagrams of a double-decker coach for two different wheel/rail contact geometries with the same equivalent conicity for the wheelset amplitude of 3 mm.

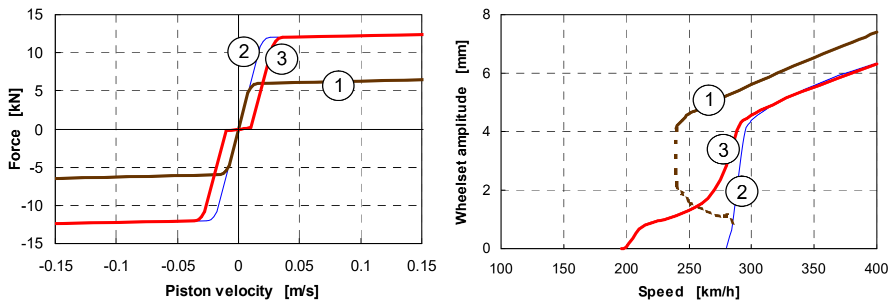

Fig. 8. Influence of yaw damper characteristic (left diagram) on the bifurcation (right diagram).

4. Discussion

The presented results demonstrate that the bifurcation analysis of a railway vehicle can lead to subcritical as well as supercritical Hopf bifurcation; the type of Hopf bifurcation is determined by the nonlinearities in the vehicle/track system, whereby the wheel/rail contact geometry has a significant influence on the vehicle behaviour at the stability limit. Can the bifurcation analysis enhance the nonlinear stability assessment in railway vehicle industry?

A detailed analysis of the bifurcation diagram for speeds far over the nonlinear critical speed can be very interesting from the theoretical point of view, however less important for industrial applications. Wheelset oscillation with a very small amplitude, say less than 1 mm would probably be overlooked during the tests due to real track irregularities, unless this oscillation is coupled with larger movements of other bodies. A very small variation in periodicity leading to a quasi-periodic or chaotic motion observed as a result of nonlinear investigations can often be more related to a particular modeling of wheel/rail contact than to real behaviour observed in service. The nonlinearity or non-smoothness is often "smoothed" in the reality as described by Piotrowski [11] on example of friction element. Detailed studies about very small deviations at particular conditions will therefore hardly enhance the stability assessment in railway industry.

For the assessment of running stability in the rolling stock industry, the main properties at various realistic conditions are of interest. From this point of view, significant differences can be observed between a system demonstrating the subcritical Hopf bifurcation and a system showing the supercritical one. What is the relationship between the bifurcation diagram and the vehicle behaviour at the stability limit? This relation is shown in Figure 9. A vehicle/track systems showing a subcritical Hopf bifurcation usually reaches the nonlinear critical speed and the safety limits at the same or similar speed. The stability assessment of such system can, however, lead to an underestimation of both criteria if the stability has been assessed by simulation applying too small disturbance or too small track irregularities. The system showing a supercritical bifurcation possesses nonlinear critical speed which is lower than the speed at which the safety limits are reached. The assessment of such system applying bifurcation analysis can deliver low critical speeds with criteria below the safety limits specified for vehicle acceptance. For a safety assessment of such system, other methods than bifurcation diagram of wheelset displacement are required, e.g. simulation of run on measured track irregularities. An exploitation of speeds higher than the nonlinear critical speed would however lead to sustaining oscillation. Even if the amplitude of this oscillation would be small, it could lead to fatigue damage and comfort deterioration. The range of speeds between the nonlinear critical speed and the speed at which the safety limits are achieved should therefore be assessed using other kind of analysis regarding the fatigue and passenger comfort.

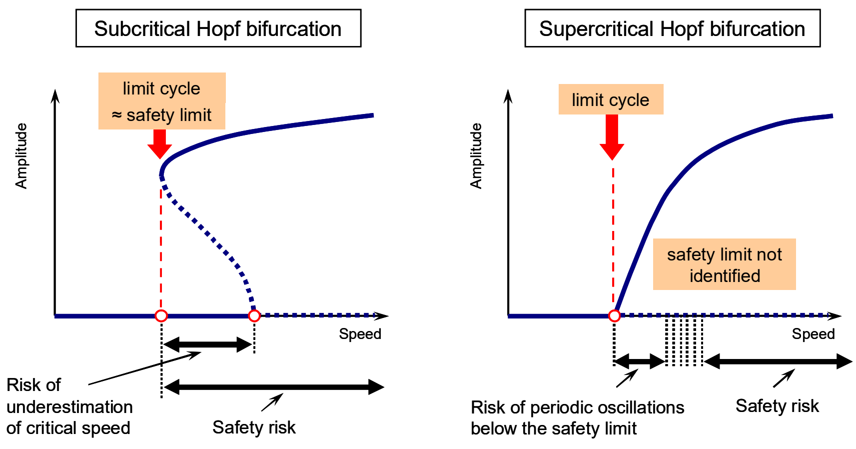

Fig. 9. Bifurcation diagram and assessment of safety and oscillation behaviour.

The bifurcation analysis allows assessing the influence of the level of track disturbance on the stability assessment and gaining a valuable output regarding the vehicle behaviour at the stability limit. It is, however, time consuming and rarely used in railway applications. Because of large variation of service conditions and parameters, a large set of investigations for different conditions is required during vehicle design in the rolling stock industry. The industry would require a robust procedure, which could be less exact, but allow a fast computation of bifurcation diagrams using a complex, realistic multi-body vehicle models for a set of different conditions wheelset/track and vehicle parameters, i.e. which allows a "rough and robust" assessment for a large set of input parameters.

5. Summary and conclusion

The paper presents typical methods used for stability analysis in the railway vehicle industry and shows that they can lead to differing critical speeds because of: deviating computation procedures, wide possible range of input conditions, and differing assessment criteria.

The bifurcation diagram computation is explained and the influence of nonlinearities of the wheel/rail contact and of the vehicle model on the type of Hopf bifurcation shown. The presented examples demonstrate in contradiction to several other publications that the bifurcation analysis of a railway vehicle can lead not only to subcritical but also to supercritical Hopf bifurcation.

The stability assessment can overestimate the critical speed of a vehicle/track system, demonstrating subcritical Hopf bifurcation if the applied disturbance is too low. Contrary, it can deliver critical speeds below the safety limits for a vehicle/track system showing supercritical Hopf bifurcation.

An application of bifurcation analysis in vehicle design and development could enhance the nonlinear stability assessment of railway vehicles. However, an efficient use of bifurcation analysis in industry is not possible today. Fast and robust algorithms or procedures applicable with commercial simulation tools would be required to allow an introduction of this method to rolling stock design and development.

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